Exercise Espionage - Solution

In general, unknown polynomials could match on arbitrarily many points, so that the problem seems impossible. Even with the restriction that the degree is at most 10, polynomials could still match on up to 10 points. The key is the constraint that $|a_i| \leq 10^5$ for all coefficients. In general, we have $P(n) = \sum_{i=0}^{d} a_i n^i$, and instead of looking at this simply as a polynomial expression, we look at it as an expansion of $P(n)$ in base n, with digits given by a_i. In general, the digits a_i could form some overlap, so that they cannot be distinguished from $P(n)$. However, as $|a_i| \leq 10^5$, if we plug in a large enough n we can distinguish the individual digits by some simple modular arithmetic. Any $n > 2 \cdot 10^5$ would work. Thus, we can determine the full polynomial $P(x)$ in a single query. Then we simply compute a root locally and send this as our second query. For this we can use the fact that any integer root of $P(x)$ has to divide a_0. If $a_0 = 0$, we know that 0 is a root. Otherwise, we simply try all divisors of a_0 locally. This is guaranteed to yield a root, as it is given that $P(n)$ has at least one integer root.